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(6) ON CERTAIN UNSTEADY MOTIONS OF A COMPRESSIBLE FLUID,

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T-57

3 February 1956

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Institute of Mechanics of the USSR Academy of Sciences

Prikladnaya Matematika i Mekhanika, Vol. 9, No 4, 1945, pp. 293 - 311

ON CERTAIN UNSTEADY MOTIONS OF A COMPRESSIBLE FLUID

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(Moscow)

~~In the present work we find a number of~~ exact solutions of the equations for one-dimensional unsteady motion of a compressible fluid are considered in the case of plane waves and motions with cylindrical and spherical symmetry.[1] The solutions are determined by ~~means of~~ simple procedures based on considerations of the theory of dimensions~~; these procedures can be considered as a general method that permits us to find in many questions of physics families of solutions which depend on arbitrary parameters~~. In concrete examples it is ~~usually~~ easy to indicate the arrangements of the problems and the hypotheses leading to solutions of the type obtained. ~~We shall apply~~ this general method is applied to both linear and nonlinear equations with partial derivatives in the most varied problems of physics and mechanics.

The method of Fourier, which is applicable to certain linear problems, is widely known. Sometimes it is possible to utilize partial solutions, which are obtained by means of the general method under consideration, as well as partial solutions obtained by the method of Fourier, for setting up the general solution of equations of motion and for the solution of boundary problems.

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[1] A short report on these solutions was published in a note by the author [1].

1. Let us consider the one-dimensional unsteady motions of a compressed fluid. Let us take time t and the linear coordinate r as independent variables. As basic unknown quantities let us take the velocity v of the particles of the fluid, the density ρ and the pressure p. The dimensions of density and pressure depend on the symbol of unit of mass; therefore the functions $\rho(r,t)$ and $p(r,t)$ must contain certain dimensional physical constants. These constants may be a part of the equations of motion, which the unknown functions satisfy, or the additional conditions by which the solution is determined (characteristics of the field occupied by the moving fluid, boundary conditions, initial conditions, etc.).

Let us consider the cases where the unknown functions contain dimensional constants, among which there are only one or two constants with independent dimensions. Special motions of such a kind may represent independent interest or may prove to be finite motions, if, in the limit, among the assigned physical characteristics only one or only two characteristics with independent dimensions are essential.

Let us assume in the beginning that there is only one dimensional constant which we denote by a. It is obvious that the dimensions of the constant a must contain the symbol of mass; therefore, without loss of generality we can consider that the dimensional formula for a is of the form

$$a = M\, L^{k}\, T^{s},$$

where k and s are constants. For the quantities v, ρ, and p we can write the formulas

$$v = \frac{r}{t} V, \quad \rho = \frac{aR}{r^{k+3} t^{s}}, \qquad p = \frac{aP}{r^{k+1} t^{s+2}} . \tag{1.1}$$

It is obvious that the abstract quantities V,R, and P canot depend on r and t and, consequently, can only be constants.[2] Consequently, the hypothesis that only one constant a is essential makes it possible, without resorting to the equations of motion, to completely determine the dependence of v, ρ, and p on r and t. If the equations of motion do not contain physical constants or contain constants with dimensions expressible in terms of the dimension a, then these equations must have solutions of the kind (1.1) when V, R, and P are constants. It is not difficult to see that in this case whatever the constants k and s may be, the motion determined by the formulas (1.1) does not contain the condition for which density and pressure are different from zero and at the same time constant throughout the entire mass of fluid.

If, besides the constant a, there is still a second constant b, then, without loss of generality, we can always assume that the dimensions of b do not contain the symbol of mass and, consequently, a formula of the form

$$[b] = L^m T^n,$$

where m and n are constants, is true; it is obvious that when $n \neq 0$ only the ratio m/n is essential.

From the determining quantities r, t, a, and b we can form only one independent abstract variable combination

$$\lambda = b\, r^{-m} t^{-n}.$$

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[2]Concerning general methods of the theory of dimensions see, for example, the author's book [2].

Turning to formula (1.1) we conclude that the abstract quantities V, R, and P can be functions of a single variable parameter λ. Further, we shall show that we can construct solutions of the type under consideration in which the functions V, R, and P or their derivatives are discontinuous.

The hypothesis that only two constants a and b with independent dimensions are essential permits us to determine the general type of distributions of the values v_o, ρ_o, and p_o, when $t = 0$. By means of the theory of dimensions [2] we find

$$v_o = \alpha_1 b^{-\frac{1}{n}} r^{\frac{m}{n}+1}, \quad \rho_o = \alpha_2 a b^{-\frac{s}{n}} r^{\frac{ms}{n}-k-3}, \quad p_o = \alpha_3 a b^{-\frac{s}{n}-\frac{2}{n}} r^{\frac{ms}{n}-k-1+\frac{2m}{n}}, \tag{1.2}$$

where α_1, α_2, α_3 are abstract quantities that we may consider as functions of r, having constant values when $r \neq 0$ and $r \neq \infty$ and having singular points when $r = 0$ and $r = \infty$ in the general case. The quantities α_1, α_2, and α_3, in particular, may be equal to zero or infinity.

If, when $t = 0$, pressure and density are constant, finite, and different from zero, then the following relations must hold:

$$\boxed{\frac{m}{n} = -1, \quad k + s = -3,} \tag{1.3}$$

from which it follows that the magnitude of the initial velocity is the same for all particles of the fluids. Discontinuity of velocity can take place only when $r = 0$ or when $r = \infty$.

Let us note certain problems for which the solution can be constructed by means of solutions of type (1.1) when

$$k = -3, \qquad s = 0, \qquad \frac{m}{n} = -1. \tag{1.4}$$

A. Classical problem of an "explosion" along a plane. It is necessary to determine the adiabatic motion of a gas, if at the initial moment of time the velocity, density, and pressure have a discontinuity along the plane $x = r = 0$, while on the right hand side for $r > 0$ we have the assigned constant values v_1, ρ_1, and p_1, and on the left hand side for $r < 0$ we have the assigned constant values v_2, ρ_2, and p_2.

As is known, equations of motion of a perfectly ideal gas do not contain dimensional constants; therefore, in this case the motion depends only on the six dimensional constants v_1, ρ_1, p_1, v_2, ρ_2, and p_2.

Among these constants there are only two with independent dimensions. We can take a and b as constants:

$$a = \rho_1 \left([a] = ML^{-3},\ k = -3,\ s = 0 \right),$$
$$b = \frac{\gamma p_1}{\rho_1} \left([b] = L^2 T^{-2},\ m = 2,\ n = -2,\ \frac{m}{n} = -1 \right). \qquad (1.5)$$

Relations (1.4) are satisfied. The solution of the assigned problem can be obtained by means of a combination of solutions of type (1.1). Consequently, in this case we can say in advance that the formulas which give expressions for v, ρ, and p in terms of r and t can depend only on the parameter $bt^2/r^2 = \lambda$. Formulas expressing v, ρ, and p in terms of r and t can change their form when passing through discontinuities[3] (weak, stationary, and strong jumps).

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[3]For an examination of possible cases of motion in this problem, see the work of N. E. Kochin [3].

B. Problem of the motion of a gas in a long cylindrical tube, closed at one end with a piston. At the first moment of time the gas is at rest, and the piston begins to move suddenly with constant velocity U. By reversing this motion, we arrive at the problem of the sudden introduction of a barrier in front of an advancing homogeneous stream of gas.

In this problem the motion is determined by the following dimensional constants: the initial density ρ_1, the initial pressure p_1, and the velocity of the piston U. We again obtain the case where the solution depends only on two constants with independent dimensions. As is known, if U is directed toward the gas, then the solution of the problem has the simplest form - - in front of the piston there is formed a zone with constant pressure, density, and velocity of gas particles equal to U. This zone is separated from the quiescent gas by a strong discontinuity, expanding with constant velocity.

C. Problem of the detonation of a gas in a cylindrical tube (detonation begins either in a plane cross section of the tube and propagates in both directions in a quiescent medium, or the detonation begins from the end of the tube closed by an immovable piston).

During detonation there is added a constant that defines the resulting specific heat of chemical reaction $q_1 - q_2$. The constant $q_1 - q_2$ in mechanical units of measurement has the dimensions of the square of velocity, which depend on the dimensions of pressure and density. Consequently, in this case we have only two dimensional constants with independent dimensions, and the solution can be formed from solutions of the type (1.1).

The solution can be generalized by assuming that the piston at the initial moment of time begins to move with a constant velocity U, or by assuming that instead of a movable piston near the end of the tube, there is created a constant pressure that is maintained with time and differs from the pressure in the quiescent gas.

For plane waves the solution of the problems enumerated is elementary. We shall examine the generalization of the specified problems in the case of cylindrical and spherical symmetry.

For the sake of concreteness we shall further restrict ourselves to a detailed study of the corresponding problems with spherical symmetry. Before that, we shall consider the following problem: At the moment $t = 0$ inside a sphere S_o of radius r_o, the gas is quiet and has a given density ρ_2 and pressure p_2; outside the sphere S_o there is the same gas, in which density, pressure, and velocity are constant and equal to ρ_1, p_1, and v_1. It is required to determine the starting motion of the gas.

It is obvious that in this case the solution depends on six dimensional constants r_o, ρ_2, p_2, ρ_1, p_1, and v_1, among which there are three constants with independent dimensions. Since the radius r_o in the general case has an essential value, the solution of this problem does not have the form (1.1). If we proceed to the limit as $r_o \rightarrow 0$, then as a result we shall obtain a problem with two independent dimensional constants, which can be solved by means of solutions of the type (1.1).

Thus we arrive at the following problems with spherical symmetry for which we shall give the methods of solution.

1° <u>All particles of homogeneous gas have the same velocity directed toward the center 0,</u> $v_1 < 0$ <u>(focussed at a point) or from the center</u> $v_1 < 0$ (<u>dispersion from center</u>).

In both cases near the point 0 there is formed a spherical expanding region of quiescent fluid, in the first case with increased pressure, in the second with reduced pressure. From the following it will be clear that there exists a critical velocity $v_1^* > 0$. If $v_1 > v_1^*$, then a vacuum forms at the center 0.

2° <u>Generalization of problem B concerning the piston</u>. When $t = 0$ the gas is quiescent, but the particles of the sphere S_o of infinitesimal radius with center at the point 0 suddenly obtain a constant velocity U directed radially from the point 0. In the following, when $t > 0$ these particles continue to move with the same constant velocity U. The fluid, as it were, is moved apart by the sphere S_o, the radius of which increases from zero proportionally to time. By means of the theory of dimensions it is not difficult to see that on the surface of the sphere S_o the pressure must be constant.

The corresponding pressure is determined from the function of the velocity U and of the characteristics of the quiescent gas. Consequently, this problem coincides with the problem of the expansion of the quiescent mass of fluid by the assigned constant pressure which arises at the point 0.

3° <u>The problem of spherical detonation</u>. When $t = 0$ at a certain point 0 in the gas, there arises a detonation that then propagates in the quiescent gas. In this case, around the point 0 an expanding sphere S_1 forms from

the quiescent gas. The density and pressure inside S_1 are constant and differ from the density and pressure of the quiescent gas through which the detonation wave[4] has not yet passed.

2. To obtain the conditions that must satisfy the values which enter formulas (1.1), it is necessary to resort to the equations of motion. It is obvious that to justify the accepted hypotheses, it is necessary that the equation of motion should not contain essential constants with dimensions independent of the dimensions a and b.

For the sake of definiteness of the problem, let us take the equations of motion of a fluid in the following form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu-1)\frac{\rho v}{r} = 0, \qquad (2.1)$$

$$\frac{\partial}{\partial t}\frac{p}{\rho^{\gamma}} + v\frac{\partial}{\partial r}\frac{p}{\rho^{\gamma}} = 0, \qquad (2.2)$$

where γ is some abstract constant. When $\nu = 1$ we have plane waves; when $\nu = 2$, cylindrical waves; and when $\nu = 3$, spherical waves.

When $\gamma = C_p/C_v$, equations (2.1) and (2.2) can be considered as the equations of the adiabatic motions of a gas, where entropy can be different for different particles of the gas. Equations (2.1) and (2.2) do not contain dimensional constants; therefore these equations must have a solution of the type (1.1), where there is only one constant a, and also in the case where there are two essential constants a and b.

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[4]The problem of spherical detonation was solved by Zeldovich [4]. Like Problem C on detonations in a cylindrical tube, the problem of spherical detonation can be complicated by the addition of the conditions of Problem 2°.

Substituting formulas (1.1) into equations (2.1) and (2.2), we obtain

$$\lambda \left[(n + mV)V' + m \frac{P'}{R}\right] = V^2 - V - (k+1)\frac{P}{R},$$
$$\lambda \left[mV' + (n+mV)\frac{R'}{R}\right] = -s - (k-\nu+3)V, \qquad (2.3)$$
$$\lambda(n+mV)\left[\frac{P'}{P} - \gamma\frac{R'}{R}\right] = -s(1-\gamma) - 2 - \left[k(1-\gamma) + 1 - 3\gamma\right]V.$$

If the constant b is absent, then the left-hand members of equations (2.3) must be replaced by zeros, as a result of which three simple finite relations are obtained connecting V, P, R, k, and s:

$$V = \frac{2}{2+\nu(\gamma-1)}, \quad P = \frac{2\nu(1-\gamma)}{(k+1)\left[2+\nu(\gamma-1)\right]^2}R, \quad s = \frac{2(\nu-k-3)}{2+\nu(\gamma-1)}. \qquad (2.4)$$

In this case formulas (1.1) give a family of exact solutions of equations (2.1) and (2.2), depending on the two arbitrary constants k and aR. Replacing t by $t - t_0$ we obtain a solution that depends on three constants.

When $\nu = 1$ in formula (1.1), the coordinate r can be replaced by $r - r_0$, after which we obtain a solution that depends on four arbitrary constants.

For an ideal gas, the entropy is expressed by the combination

$$\frac{p}{\rho^\gamma} = a^{1-\gamma}\frac{P}{R^\gamma} r^{k(\gamma-1)+3\gamma-1} t^{s(\gamma-1)-2},$$

which in the general case is different for different particles of fluid. If

$$k = -\frac{3\gamma-1}{\gamma-1}, \qquad s = \frac{2}{\gamma-1},$$

then for the resulting family of solutions that depend on one constant, the entropy is identical for all particles of the fluid.

When $m \neq 0$, integration of the system of equations (2.3) can be reduced to an integration of a single ordinary differential equation of the first order,

$$\frac{dz}{dV} = \frac{z\left\{\left[2(V-1)+\nu(\gamma-1)V\right](V-q)^2-(\gamma-1)V(V-1)(V-q)-\left[2(V-1)+x(\gamma-1)\right]z\right\}}{(V-q)\left[V(V-1)(V-q)+(x-\nu V)z\right]} \qquad (2.5)$$

where the following designations are used:

$$z = \frac{\gamma P}{R}, \qquad q = -\frac{n}{m}, \qquad x = \frac{s+2+q(k+1)}{\gamma} .$$

After equation (2.5) has been integrated, the dependence of V, R on λ will be determined by means of quadratures from the relations

$$\frac{d\ \ln \lambda^{\mu}}{dV} = \frac{(V-q)^2-z}{V(V-1)\ (V-q)+(x-\nu V)z}, \qquad (2.6)$$

$$\left(\mu = \frac{1}{m}\right)$$

$$(V-q)\,\frac{d\ \ln R}{d\ \ln \lambda^{\mu}} = \frac{V(V-1)\ (V-q)+(x-\nu V)z}{z-(V-q)^2} - \left[s+(k-\nu+3)V\right]. \qquad (2.7)$$

In the general case the integration of equation (2.5) must be performed approximately. When $m = 0$, i.e., when $q = \infty$, equation (2.5) is easily integrated, and we obtain the integral

$$\frac{\gamma P}{R} = z = \gamma A V^2(V-1)^{\nu(\gamma-1)}, \qquad (2.8)$$

where A is a constant of integration. Assuming $n = 1$ (i.e., $\lambda = b/t$, where b is the characteristic time), we find from equation (2.3) that

$$d\ \ln \lambda = \frac{dV}{V\left[V-1-A(k+1)V(V-1)^{\nu(\gamma-1)}\right]}, \qquad (2.9)$$

$$d\ \ln R\,\lambda^{s} = -\frac{(k+3-\nu)dV}{V-1-A(k+1)\ V(V-1)^{\nu(\gamma-1)}} . \qquad (2.10)$$

In this case the solution of the equations of motion of the fluid (2.1) and (2.2) can be written in the form

$$\frac{\lambda}{\lambda_o} = \frac{t_o}{t} = \exp\int \frac{dV}{V\left[(V-1)-A(k+1)V(V-1)^{\nu(\gamma-1)}\right]}, \tag{2.11}$$

$$\rho = \frac{aB}{r^{k+3}} \exp\left\{-(k+3-\nu)\int \frac{dV}{(V-1)-A(k+1)V(V-1)^{\nu(\gamma-1)}}\right\}, \tag{2.12}$$

$$p = \frac{r^2}{t^2} AV^2(V-1)^{\nu(\gamma-1)}\rho, \quad v = \frac{r}{t} V .$$

These formulas give an exact solution of equations (2.1) and (2.2) which depend on the four arbitrary independent constants $\underline{t_o}$, $\underline{A}$, $\underline{aB}$ and $\underline{k}$.

For some particular values of γ the integrals entering these formulas can be calculated in a simple form.

If $k = -1$, then for any γ the integrals are easily calculated, after which we obtain the following exact solution of the equations (2.1) and (2.2):

$$v = \frac{r}{t-t_o}, \qquad \rho = \alpha\frac{(t-t_o)^{2-\nu}}{r^2}, \qquad p = \beta \frac{1}{(t-t_o)^{\nu\gamma}}, \tag{2.13}$$

where t_o, α, and β are arbitrary constants. In the motions described by the formula (2.13) pressure depends on time, but is constant throughout the entire mass of fluid. All particles move uniformly and rectilinearly, but with different velocities. For each particle of the fluid the entropy is constant, but is different for different particles. Where the symmetry is cylindrical the distribution of density is stationary. A more general solution is obtained if in the solution (2.13) the constant α is replaced

by $F\left(\frac{r}{t-t_o}\right)$, where F is an arbitrary function.

Solution (2.13) coincides in form with the solution furnished by formulas (1.1) when V, R and P are constants, determined by formulas (2.4), but differs from it by the fact that it depends on the two dimensional constants α and β which in the given case enter in a special form.

When $\nu = 1$ in formulas (2.13), the coordinate r can be replaced by $r-r_o$, after which we obtain a solution depending on four arbitrary constants; this solution differs from the corresponding solution that satisfies formula (2.4).

3. Solutions in the form (2.1) can be sought when there are surfaces of weak or strong discontinuities. These characteristic surfaces will satisfy certain values of the coordinate $r = r^*$, which depends on time.

The dynamic conditions connecting the values v_1, ρ_1, p_1 and v_2, ρ_2, p_2 on different sides of surfaces of strong jumps have the form:

Condition of constancy of mass

$$\rho_1(v_1-c) = \rho_2(v_2-c), \tag{3.1}$$

where c is the velocity of displacement of the jump; and

Condition of conservation of momentum

$$\rho_2(v_2-c)\ (v_1-v_2) = p_2-p_1 . \tag{3.2}$$

We shall write the equation of energy assuming that the gas is ideal:

$$q_1 - q_2 + \frac{\gamma_1\, p_1}{(\gamma_1-1)\rho_1} - \frac{\gamma_2\, p_2}{(\gamma_2 - 1)\rho_2} + \frac{1}{2}\left[(v_1-c)^2 - (v_2-c)^2\right] = 0, \tag{3.3}$$

where $q_1 - q_2$ denotes the energy referred to a unit of mass, which can be generated by the chemical reactions accompanying the jump. It is necessary to introduce the quantity $q_1 - q_2$ by considering the phenomenon of detonation. We shall assume that the difference $q_1 - q_2$ is expressed in mechanical units of measurement and therefore the dimensions of $q_1 - q_2$ are equal to the square of the velocity. The constant γ may be different on different sides of the surface of discontinuity.

In the jumps t, a and b will be the dimensional determining parameters from which it is impossible to form a dimensionless combination; therefore, for solutions of (1.1) on the jumps, the relations

$$\lambda = \lambda_o = \text{const}, \qquad r^* = \lambda_o^{-\frac{1}{m}} b^{\frac{1}{m}} t^{-\frac{n}{m}} \tag{3.4}$$

will hold.

Since $c = dr^*/dt$, then

$$c = -\frac{n}{m}\frac{r^*}{t}. \tag{3.5}$$

We can further assume

$$q_1 - q_2 = q\left(\frac{r^*}{t}\right)^2, \tag{3.6}$$

where λ_o and $\underline{q}$ are abstract constants.

Solutions of the form considered can exist during the generation of heat by the chemical reactions, if this generation can be subordinated to a law satisfying the formula (3.6).

Motion where there are surfaces of discontinuity is impossible, if there is only one essential dimensional constant $\underline{a}$.

As pointed out above in (1.3), if motion starts from a state of rest in which the fluid is homogeneous, then $m/n = -1$. Consequently, for motions

of the type considered that starts from the state of rest, in which p_o and ρ_o are not equal to zero or infinity, it follows that the velocity of the shock waves is constant and the specific heat of the reaction $q_1 - q_3$ must also be a constant.

Substituting formulas (1.1) in the relations (3.1), (3.2), and (3.3) and introducing the designations

$$\frac{v-c}{r^*/t} = u, \qquad \frac{\gamma P}{R} = \frac{\gamma p}{\rho}\left(\frac{r^*}{t}\right)^2 = z , \tag{3.7}$$

we obtain

$$\frac{R_1}{R_2} = \frac{u_2}{u_1} , \tag{3.8}$$

$$\frac{z_1}{\gamma_1 u_1} + u_1 = \frac{z}{\gamma_2 u_2} + u_2, \tag{3.9}$$

$$q + \frac{z_1}{\gamma_1 - 1} + \frac{u_1^2}{2} = \frac{z_2}{\gamma_2 - 2} + \frac{u_2^2}{2} . \tag{3.10}$$

From equations (3.9) and (3.10) it is easy to express z_2 and u_2 in terms of z_1, u_1 and q.

For shock waves without chemical reactions ($\gamma_1 = \gamma_2 = \gamma$ and $q = 0$), two solutions of equations (3.9) and (3.10) have the simple form

$$u_1 = u_2, \quad z_2 = z_1,$$

$$u_2 = u_1\left[1 + \frac{2}{\gamma+1} \; \frac{z_1 - u_1^2}{u_1^2}\right] , \tag{3.11}$$

$$z_2 = \left(\frac{\gamma-1}{\gamma+1}\right)^2 \frac{1}{u_1^2}\left[u_1^2 + \frac{2z_1}{\gamma-1}\right]\left[\frac{2\gamma}{\gamma-1} u_1^2 - z_1\right] \tag{3.12}$$

Since equations (3.9) and (3.10) are symmetrical with respect to z_1, V_1 in terms of z_2, V_2 it is sufficient to transpose the indices 1 and 2 in formulas (3.11) and (3.12)[5].

In the following, we shall assume that the index 1 corresponds to that side of the jump to which we cross during the growth of r.

For a general analysis, another relation is useful:

$$z_2 - u_2^2 = -(z_1 - u_1^2)\left(1 + \frac{2}{\gamma+1}\,\frac{z_1 - u_1^2}{u_1^2}\right). \qquad (3.13)$$

The value of z in its physical sense is essentially positive. According to the theorem of Tsemplen, the inequality $V_1 - V_2 < 0$ must be satisfied, which yields

$$u_1 - u_2 = V_1 - V_2 < 0. \qquad (3.14)$$

On the half-surface $-\infty < V < +\infty$, $r > 0$ (Fig. 1) are marked the

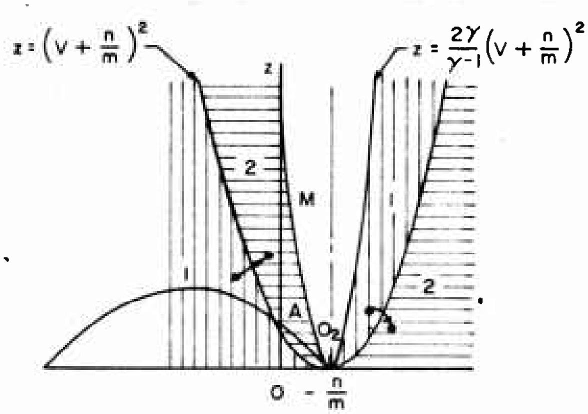

Fig. 1

[5]In the preceding considerations, formulas (2.1) are utilized for formulating conditions in the jump in the dimensionless form (4.8), (4.9), and (4.10) on page 25; therefore the relations mentioned are true for any motions of a gas with straight jumps. For special motions of the type considered, we have $u = V + n/m$.

regions that transform into each other by means of relations (3.11) and (3.12).

The straight line $z_1 = 0$ transforms into the parabola $z = 2\gamma(V+n/m)^2/(\gamma-1)$. The points of the parabola $z = (V+n/m)^2$ will transform into themselves.

From the theorem of Tsemplen it follows that the points z_1, V_1, must be located in the regions that are hatched vertically, and the corresponding values of z_2 and V_2 will be found in the regions that are hatched horizontally. Possible transitions from z_1, V_1 to z_2, V_2 are indicated by arrows.

Points of the type M, located above the parabola $z = 2\gamma(V+n/m)^2/(\gamma-1)$, cannot correspond to the boundaries of the jumps.

Let us now consider in more detail another case where the condition of rest is a particular solution in the family under consideration. In this case

$$-\frac{n}{m} = 1, \quad c = \frac{r^*}{t}; \quad z = \frac{a^2}{c^2}, \quad V-1 = \frac{v-c}{c} \left(a = \sqrt{\frac{\gamma p}{\rho}} \right)$$

where a is the velocity of sound.

If $z > (V - 1)^2$, then the velocity of the fluid particles with respect to the jump is less than the velocity of sound; when $z < (V - 1)^2$, it is greater than the velocity of sound. On the parabola $z = (V - 1)^2$ the relative velocity of the particles is equal to the velocity of sound.

If $v = 0$ when $r \neq 0$ and $t \neq \infty$, then $V = 0$; whence it follows that the points on the z-axis correspond to the conditions of rest.

On the straight line $V = 1$ we have $v = r/t$. This means that the velocity of the fluid particles is equal to the velocity of transition of the conditions of motion λ = constant that satisfy the points of this straight line. Consequently, if there are stationary discontinuities in the stream, then they must satisfy the points of the straight line $V = 1$.

Let us assume that we have a condition of rest on side I; this condition can change into motion by a jump only for points of the z-axis located on the segment O_1A, for which $z_1 = a_1^2/c^2 \leq 1$. The points of the segment O_1A transform by the jump into the points of the arc of the parabola AO_2, the equation of which has the form

$$z_2 = (1-V_2)\ (1+ \frac{\gamma-1}{2} V_2)\ . \tag{3.15}$$

The point O_2 for which $z_2 = 2\gamma(\gamma-1)/(\gamma + 1)^2$, $V_2 = 2/(\gamma + 1)$ corresponds to the point $O_1(z_1 = 0,\ V_1 = 0)$. If the velocity of sound in a quiescent medium is finite, then the limiting case when the speed of propagation of the jump is equal to infinity corresponds to the jump from O_1 to the point O_2. The velocity of propagation that is equal to the velocity of sound corresponds to the point A $(z = 1,\ V = 0)$. When the point z_1, V_1 approaches the point A, the jumps in velocity, density, and pressure converge toward zero. Only a weak discontinuity may correspond to the point A. By the position of the point A on the segment O_1A the velocity of propagation of the jump into the quiescent fluid is determined.

In the case of the propagation of a direct condensation jump in a quiescent gas $v_1 = 0$ (when $q_1 - q_2 = 0$ and $\gamma_1 = \gamma_2 = \gamma$), the relations (3.1), (3.2), and (3.3) can be written in the form

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}\left(\frac{c}{a_1}\right)^2 + \frac{1-\gamma}{1+\gamma}\,, \tag{3.16}$$

$$\frac{\rho_1}{\rho_2} = \frac{\gamma+1}{\gamma-1+2(a_1/c)^2}\,, \tag{3.17}$$

$$\frac{v_2}{c} = \frac{2}{\gamma+1}\left[1-\left(\frac{a_1}{c}\right)^2\right]. \tag{3.18}$$

These formulas are convenient for evaluating the elements of a jump by means of the velocity of propagation of the jump or by means of the velocity of the particles of the gas behind the jump v_2. For a better illustration, the relations represented by the formulas (3.16), (3.17), and (3.18) are shown graphically by solid lines in Fig. 2.

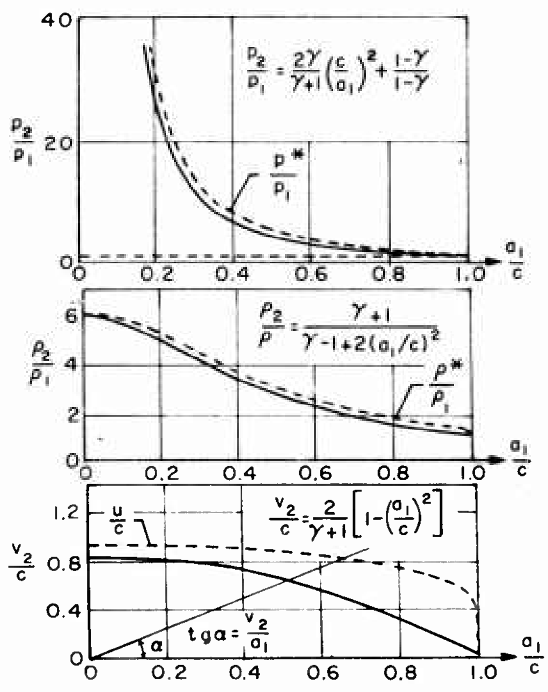

Fig. 2

If the ratio v_2/a_1 is assigned, then in order to determine the ratio a_1/c it is sufficient to draw a straight line with a slope equal to v_2/a_1 through the origin of coordinates on the lower graph. The intersection of this straight line with the parabola (3.18) determines the ratio a_1/c.

We shall now examine another case, when $q_1 - q_2 \neq 0$ and $\gamma_1 \neq \gamma_2$, and the shock wave propagates in a quiescent medium $V_1 = 0$ or $a_1 = -1$.

In this case equations (3.9) and (3.10) assume the form

$$z_2 + \gamma_2 \left(\frac{a_1^2}{\gamma_1 c_1^2} \right) u_2 + \gamma_2 u_2^2 = 0, \tag{3.19}$$

$$\frac{z_2}{\gamma_2 - 1} + \frac{u_2^2}{2} = \frac{1}{2} + \frac{1}{\gamma_1 - 1} \frac{a_1^2}{c^2} + \frac{q_1 - q_2}{c^2}. \tag{3.20}$$

When $\underline{a_1}^2$, $\underline{q_1} - \underline{q_2}$, and $\underline{c}^2$ are constants in the z, u plane, these equations are represented by parabolas located as shown in Fig. 3, if

$$(\gamma_2 - 1) \frac{q_1 - q_2}{a_1^2} \geq \frac{\gamma_1 - \gamma_2}{\gamma_1(\gamma_1 - 1)}. \tag{3.21}$$

The equality sign corresponds to the case where the parabolas intersect at the point D. If the inequality (3.21) is not satisfied, then the system (3.19) and (3.20) has two solutions - for one of them $V_2 < 0$, and for the other $1 > V_2 > 0$.

For the existence of two real dissimilar solutions of the equations (3.19) and (3.20), the constants $\underline{a_1}$, $\underline{c}$, and $\underline{q_1} - \underline{q_2}$ must satisfy the relation

$$\left(1 + \frac{1^2}{\gamma_1 c^2}\right)^2 > 2\,\frac{\gamma_2^{\,2} - 1}{\gamma_2^{\,2}}\left[\frac{1}{2} + \frac{1}{\gamma_1 - 1}\,\frac{a_1^{\,2}}{c^2} + \frac{q_1 - q_2}{c^2}\right]. \tag{3.22}$$

The roots of equations (3.19) and (3.20) converge when in the relation (3.22) the sign of the inequality is changed by the sign of equality; in this case we arrive at the equation

$$\left(\frac{c}{a_1}\right)^4 - 2(\gamma_2^{\,2} - 1)\left[\frac{q_1 - q_2}{a_1^{\,2}} - \frac{\gamma_1 - \gamma_2^{\,2}}{\gamma_1(\gamma_1 - 1)(\gamma_2^{\,2} - 1)}\right]\left(\frac{c}{a_1}\right)^2 + \frac{\gamma_2^{\,2}}{\gamma_1^{2}} = 0, \tag{3.23}$$

and for the roots of equations (3.19) and (3.20) we obtain at the same time the expressions

$$u_2' + u_2'' = -\frac{\gamma_2}{\gamma_2 + 1}\left(1 + \frac{a_1^{\,2}}{\gamma_1 c^2}\right),\quad z_2' = z_2'' = \frac{\gamma_2^{\,2}}{(\gamma_2 + 1)^2}\left(1 + \frac{a_1^{\,2}}{\gamma_1 c^2}\right)^2. \tag{3.24}$$

It is obvious that the corresponding solution determines the point located on the parabola

$$z_2 = (V_2 - 1)^2. \tag{3.25}$$

If the inequality (3.22) exists, then two solutions are obtained for which the corresponding points M and N (Fig. 3) are located on different sides of the parabola (3.25).

It is not difficult to satisfy oneself that if the ratio $(q_1 - q_2)/a_1^{\,2}$ is assigned in such a way that the inequality (3.18) is satisfied, then for different values of $c^2/a_1^{\,2}$ the corresponding points M and N will be located on a certain third order curve, while equation (3.23) is satisfied in the points of the parabola (3.25), and the quantity

c^2/a_1^2 attains an extremum. The largest root of equation (3.23) corresponds to a minimum, and the smallest to a maximum of the ratio c^2/a_1^2.

If in relation (3.21) there is the sign of equality, then equation (3.23) has the form

$$\left(\frac{c_o^2}{a_1^2} - \frac{\gamma_2}{\gamma_1}\right)^2 = 0 \qquad (3.26)$$

It is obvious that in this case the velocity c_o is equal to the velocity of sound in a gas with the adiabatic coefficient γ_2. From formulas (3.24) for the corresponding values of V_2 and z_2 we find $V_2 = 0$, and $z_2 = 1$.

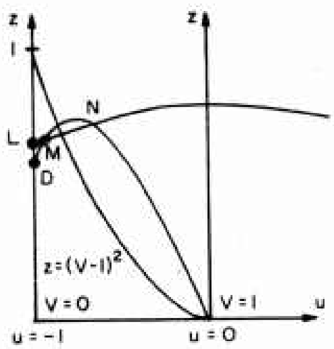

Fig. 3

If in the relation (3.21) there is a sign of inequality, then the equations (3.24) have two roots c_1 and c_2 that satisfy the inequality $c_1 < c_o < c_2$.

All the reasonings of this paragraph are applicable to motions with plane waves and to motion with cylindrical and spherical symmetry.

4. Let us now consider the question of determining all the solutions when $m = 2$, $n = -2$, $s = 0$, $k = -3$, and $\nu = 3$. Let us assume

$$a = \rho_1, \quad b = \frac{\gamma p_1}{\rho_1} = a_1^2, \quad \lambda = a_1^2 \frac{t^2}{r^2},$$

where p_1 and ρ_1 are certain selected values of pressure and density.

Equations (2.5) and (2.6) in the case under consideration assume the form

$$\frac{dz}{dV} = 2\frac{z}{V}\frac{z-(V-1)(\gamma V-1)}{3z-(V-1)^2}, \tag{4.1}$$

$$\frac{1}{2}\frac{d\ \ln\lambda}{dV} = \frac{z-(V-1)^2}{V\left[3z-(V-1)^2\right]}. \tag{4.2}$$

Moreover, when $V \neq 1$ and the last of equations (2.3) is integrated, we obtain the following connection between P, R, and λ:

$$P = \frac{R^{\gamma}\lambda}{\gamma\beta^{\gamma-1}}, \tag{4.3}$$

where β is a constant of integration.

Using the relation (4.3) and the designation $\gamma P/R = z$, we can write formulas (1.1) in the form

$$v = a_1\frac{V}{\sqrt{\lambda}}, \qquad \rho = \rho_1\beta\left(\frac{z}{\lambda}\right)^{\frac{1}{\gamma-1}}, \qquad p = p_1\beta\left(\frac{z}{\lambda}\right)^{\frac{\gamma}{\gamma-1}}. \tag{4.4}$$

In regions of continuous motion the quantity p/ρ^{γ} of the various fluid particles will be identical, since

$$\frac{p}{\rho_1{}^{\gamma}} = \frac{p_1}{\rho_1{}^{\gamma}} \frac{1}{\beta^{\gamma-1}} \quad . \tag{4.5}$$

It is obvious that the abstract constant β is closely connected with the value of entropy. If, at a certain point in the region of continuous motion, pressure and density have the values p_1 and ρ_1, then it is obvious that in this region $\beta = 1$.

The solution expressed by formulas (4.4) depends on four arbitrary constants. Two of them are $\rho_1\beta$ and $p_1\beta$; two others will enter during the integration of the differential equations (4.1) and (4.2).

The main difficulty lies in the integration of equation (4.1). After the integration of this equation, the parameter λ is determined as a function of V from equation (4.2) by means of simple quadrature.

Let us consider the behavior of the integral curves of equation (4.1) in the zV plane. This equation has five special points:

$O(0,0)$, $A(1,0)$, $B(z^*,V^*)$, $C(0,1)$, $D(\infty, \pm\infty)$,

while

$$z^* = \frac{3(\gamma-1)^2}{(3\gamma-1)^2}, \qquad V^* = \frac{2}{3\gamma-1} \quad .$$

The points O, A, and C are nodes and the points B and D are saddles. The general form of the integral curves in the half-plane z > 0 is shown in Fig. 4.

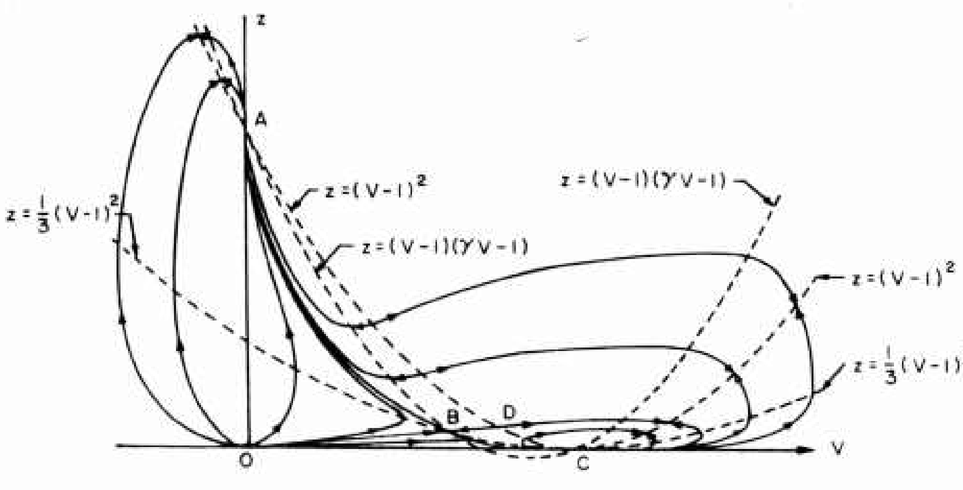

Fig. 4

In the points of the parabola $z = (V-1)(\gamma V-1)$, the integral curves have tangents parallel to the V-axis; in the points of the parabola $z = \frac{1}{3}(V-1)^2$, they have tangents parallel to the z-axis.

Near the point 0, the solutions of equations (4.1) and (4.2) have the form

$$z = k_1 V^2 + \ldots, \qquad V = \frac{\sqrt{\lambda}}{k_2} + \ldots, \tag{4.6}$$

where $k_1 > 0$ and $k_2 \gtrless 0$ are arbitrary constants. $\lambda = a_1{}^2 t^2/r^2 = 0$ corresponds to the point 0; consequently, the infinitely remote point $r = \infty$ corresponds to the point 0 in the space of the motion of the fluid.

At infinity for velocity, density, and pressure the values v_o, ρ_o and p_o are determined by the formulas

$$v_o = \frac{a_1}{k_2} \quad \left(k_2 = \frac{a_1}{v_o}\right), \qquad \rho_o = \rho_1 \beta \left(\frac{k_1}{k_2{}^2}\right)^{\frac{1}{\gamma-1}}, \quad p_o = p_1 \beta \left(\frac{k_1}{k_2{}^2}\right)^{\frac{\gamma}{\gamma-1}}. \tag{4.7}$$

If $\rho_o = \rho_1$, $p_o = p_1$, then $\beta = 1$, $k_1 = k_2{}^2$.

In the z, V plane near the point 0, the integral curves have the form of parabolas. As the velocity increases to infinity the corresponding parabolas approach the V axis. If $v_o > 0$, then the shifting upward along the right-hand branch of the parabola corresponds to the approach from infinity to the center of symmetry; if $v_o < 0$, then the shifting upward along the left-hand branch of the parabola corresponds to the approach from infinity to the center of symmetry.

Equations (4.1) and (4.2) have an exact solution:

$$z = 0, \qquad V = \frac{\sqrt{\lambda}}{k_2} \tag{4.8}$$

The conditions of motion in which either $p = 0$ or $\rho = \infty$ correspond to this solution.

Moreover, equations (4.1) and (4.2) have the solution

$$V = 0, \qquad z = k_3\lambda . \tag{4.9}$$

It is obvious that this solution corresponds to the state of rest

$$v = 0, \qquad \rho = \rho_1 \beta k_3^{\frac{1}{\gamma-1}} \qquad p = p_1 \beta k_3^{\frac{\gamma}{\gamma-1}} \tag{4.10}$$

If ρ_1 and p_1 are the density and pressure corresponding to the state of rest, then $\beta = 1$, $k_3 = 1$.

The integrated straight line satisfies the state of rest. The moving points of the fluid for which $\lambda = a_1^2 t^2/r^2$ = constant correspond to the fixed points of this straight line.

It is not difficult to see that to the point $z = 1$, $V = 0$ there corresponds a state of motion which propagates in the fluid with a velocity equal to the velocity of sound satisfying this state.

Indeed, from formulas (4.9) and (4.10) when $z = 1$ we obtain

$$\frac{r^2}{t^2} = a_1^2 k_3 = \frac{\gamma p}{\rho} .$$

When $z < 1$, the velocity of propagation of the corresponding states in the quiescent fluid is greater than the velocity of sound, and when $z > 1$ it is less than the velocity of sound.

Let us consider the behavior of the integral curves near the point A for which $z = 1$, $V = 0$. Near this point the equation of the first approximation for equation (4.1) has the form

$$\frac{dz}{dV} = \frac{z-1 + (1+\gamma)V}{V} . \tag{4.11}$$

The general integral of equation (4.11) is represented by the formula

$$z-1 = CV + (1+\gamma)V \ln V, \tag{4.12}$$

where C is a constant of integration.

It is obvious that at the point A there is a node, and that the integral curves converge to the point A, tangent to the z axis.

It is not difficult to see that in the motion along the integral curve, the point A is reached for a finite value of the parameter λ, since at the point A we have

$$\frac{dV}{d\lambda} = 0, \qquad \frac{1}{\lambda}\frac{d\lambda}{dz} = 1 . \tag{4.13}$$

The point A may correspond to a weak discontinuity. The state of rest, given by the straight line V = 0, may transform continuously into motion determined by other integral curves converging in the point A.

The special point B is a saddle. Through the point B pass two integral curves with slopes

$$\frac{dz}{dV} = \frac{\gamma-1}{2(3\gamma-1)}\left[\gamma-3\pm\sqrt{(\gamma-3)^2+4(\gamma-1)(3\gamma+1)}\right] . \tag{4.14}$$

During the motion to the point B along the integral curves corresponding to the plus sign, the parameter λ tends toward $+\infty$; during motion to the point B along the integral curves corresponding to the minus sign, the parameter λ tends toward zero.

A special solution satisfies the point B:

$$v = \frac{2}{3\gamma-1}\frac{r}{t}, \quad \rho = \rho_1\, \beta z^{*\,\frac{1}{\gamma-1}}\left(\frac{r}{a_1 t}\right)^{\frac{2}{\gamma-1}}, \quad p = p_1\, \beta z^{*\,\frac{\gamma}{\gamma-1}}\left(\frac{r}{a_1 t}\right)^{\frac{2\lambda}{\gamma-1}} \tag{4.15}$$

The solution (4.15) is a particular case of the dependence of the general solution on the one-dimensional constant which we examined in paragraphs 1 and 2. Formulas (1.1) and (2.4) become formulas (4.15) when

$$k = \frac{3\gamma-1}{1-\gamma}, \qquad s = \frac{2}{\gamma-1}. \tag{4.16}$$

In the solution of (4.15), when $t = 0$, $r \neq 0$, we have $v = \infty$, $\rho = \infty$, and $p = \infty$; when $t > 0$, the infinite values are maintained only when $r = \infty$; when $r = 0$, we have $v = 0$, $\rho = 0$, $p = 0$.

Let us consider, finally, the special point C. Near the point $V = 1$, $z = 0$ for sufficiently small $z > \frac{1}{3}(V-1)^2$, and the continuation of the integral curves to the right and left lead to the intersection with the parabola $z = \frac{1}{3}(V-1)^2$. In the points of this parabola we have $(dz/DV) = \infty$; during further continuation the integral curves tend toward the point C, remaining below the parabola $z = \frac{1}{3}(V-1)^2$. Consequently the point C is a node.

It is not difficult to show that for all values of $\gamma > 1$ the following inequality exists:

$$\lim_{V\to 1} \frac{z}{(V-1)^2} < \frac{1}{3}. \tag{4.17}$$

From this it follows that the derivative $d \ln \lambda/dV$ has a finite value when $V = 1$. Consequently, during the motion along the integral curve we arrive at the point C with a finite value of the parameter λ. The point C may correspond to a stationary discontinuity or to a boundary which is shifting together with the fluid particles.

At the point C we have $z = \gamma P/R = 0$, and from this and from (4.3) it follows that for all solutions different from $z = 0$, during the approach to the point C pressure and density tend toward zero.

Motion along the integral curve on the side of increase in parameter λ corresponds to a decrease in r. When $0 \leq r \leq \infty$, we have $\infty \leq \lambda \leq 0$.

By means of equation (4.2) it is easy to show the directions of increase in parameter λ along the integral curves. In Fig. 4 these directions are shown by arrows.

On the parabola $z = (V-1)^2$, the parameter λ reaches a maximum when $V < 0$ and $V > 1$ and a minimum when $0 < V < 1$.

It is obvious that the continuous transition along the integral curve through the points of this parabola, excluding the points A and C, is physically inadmissible, since it leads to non-singlevaluedness.

In the center of symmetry of the stream, $r = 0$ and $\lambda = \infty$. In the zV plane, the motion of the fluid continues to the center $r = 0$ along the integral straight line $V = 0$, $z \to +\infty$, satisfying the state of rest; along the straight line $z = 0$, $V \to \pm \infty$, for which either the density is infinite or the pressure is equal to zero; for the special solution (4.15), i.e., for the point B, and finally, for the motion which satisfies the integral curves OB and DB, resting on a special solution for the point B.

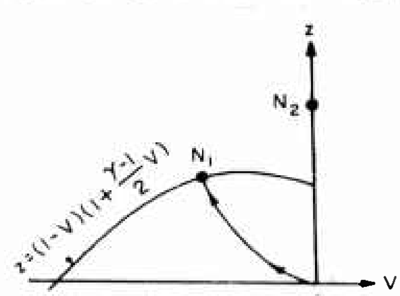

Fig. 5

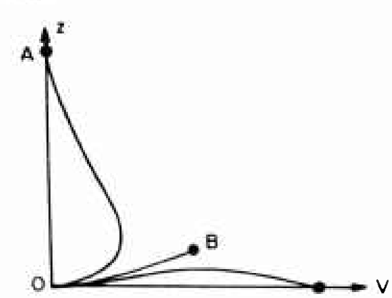

Fig. 6

At an infinitely remote point of the stream we have $r = \infty$, $\lambda = 0$. In the z_1V plane only the special points O and B can correspond to the infinitely remote point of the stream. For the point B, the velocity is infinite at infinity. For solutions starting from the point O, the velocity of the fluid at infinity can have any value.

5. Based on the general analysis, given above, of the conditions at the jumps and the behavior of the integral curves of equations (4.1) and (4.2) in the z_1V plane, it is not difficult to construct solutions of the problems pointed out in Section 1 (page 2).

Problem 1°. The form of the integral curve when $v_1 < 0$ is shown in Fig. 5. The initial data for the integral curve are determined by the values $v_1 = v_o$, $\rho_1 = \rho_o$, and $p_1 = p_o$ by means of formulas (4.7) and (4.8).

At the point N_1 of the intersection of the parabola $z = (1-V)\left[1+(\gamma-1)V/2\right]$ with the integral curve, a strong jump arises, beyond which the fluid is at rest (point N_2). The velocity of propagation of the jump C, which is equal to the velocity of growth of the radius of the spherical nucleus with the stationary gas, is determined by the value of the parameter $\lambda^* = \lambda P_1/(\rho_1 c^2)$, satisfying the points N_1 and N_2.

If $v_1 > 0$, then the corresponding integral curves are represented in Fig. 6. For sufficiently small values of v_1, integral curves of the type OA are obtained, the motion being continuous for all values of r. Near the center of symmetry a spherical nucleus of quiescent gas

forms that transforms by a weak discontinuity into motion, and the point A corresponds to the weak discontinuity. At a certain initial velocity $v_1 = v_1^*$, the integral curve OB is formed in the z_1V plane. The point B corresponds to the center of symmetry; in this case the velocity of the gas will be equal to zero only at one point - the center of symmetry. If $v_1 > v_1^*$, then the integral curves converge toward the point C, at which the parameter λ has a certain finite, nonzero, constant value λ^*, and pressure and density become zero. Consequently, at the initial velocities $v_1 > v_1^*$ in the gas a vacuum is formed, expanding with constant velocity determined by the value λ^*.

Problem 2^o. Let us designate by U the velocity of a spherical piston which is equal to the velocity of the gas particles on the sphere S_o; the pressure inside this sphere is constant and equal to p^*.

We designate by ρ_1 and p_1 the density and pressure in the quiescent fluid. In the zV plane a certain point of the straight line V = 1 corresponds to the bounding sphere S_o. Integral curves, starting from points of the straight line V = 1, intersect the parabola $z = (V-1)^2$. Therefore, continuation of motion to the point O, which corresponds to an infinitely remote point, is possible only by a jump.

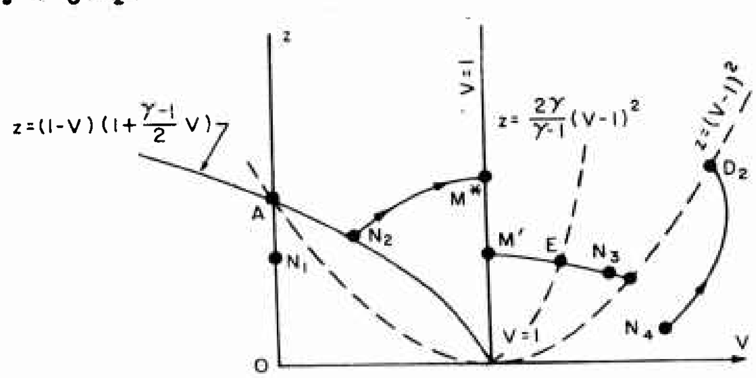

Fig. 7

Assuming that the motion of the fluid starts from the state of rest, it follows that on the external side of the jump $V_1 = 0$, $z_1 \neq 0$, i.e., the corresponding point is located on the z-axis, whence it follows that the point $N_2(z_2, V_2)$ is determined by the intersection of the integral curve (Fig. 7) with the parabola

$$z = (1-V)(1 + \frac{\gamma-1}{2} V).$$

On the straight line $V = 1$, the point M^* corresponds to the sphere S_o and the value

$$\lambda^* = \frac{\gamma p_1}{\rho_2 U^2} > \lambda_1 = \frac{\gamma p_1}{\rho_1 c^2},$$

so that $c > U$.

When $c^2 \to a_1^2$, the values z^* and λ^* tend to infinity, and the velocity U tends to zero.

As a result of numerical calculations, we have determined the dependence of the ratios U/c, ρ^*/ρ_1, p^*/p_1 on the quantity a_1/c. In Fig. 2 these dependencies are represented in the form of dashed curves; comparison of the dashed curves with the solid curves gives an increase in the ratios under consideration at the transition from the jump to the piston (sphere S_o).

The examined motion may be continued inside the sphere S_o. During the continuation the sphere S_o may become a surface of stationary discontinuity (only density is discontinuous; velocity and pressure are continuous). On the straight line $V = 1$, the integral curve $z = z(V)$ may have a point of discontinuity. The magnitude of

discontinuity (location of the points M^* and M') is determined by the ratio of the densities on the different sides of the sphere S_o, which may be given arbitrarily.

From the condition of singlevaluedness, it follows that from the point M' it is possible to continue the integral curve to the right (see Fig. 7) only to the point D_1 of the intersection with the parabola $z = (V-1)^2$. After the intersection of the integral curve with the parabola $z = 2\gamma(V-1)^2/(\gamma-1)$ at the point E, further continuation of motion may be realized by a jump from a certain point N_3 of the segment ED to the corresponding point N_4. From the point N_4 the integral curve may be continued only when the conditions of singlevaluedness to the intersection of the parabola $z = (V-1)^2$ at the point D_2 are satisfied. According to the theorem of Tsemplen no other jumps can occur. The velocities of propagation c, U, c', and U' correspond to the points N_1, N_2; M^*M'; N_3, N_4, and D_2. The correctness of the inequalities $c > U > c' > U'$ is obvious.

Consequently, the motion can be continued inside S_o, but this continuation can be realized only up to a certain sphere S, the radius of which increases according to the law $r = U't$. On the surface of the sphere S, the density, pressure, and velocities of the fluid (greater than U') are constant.

Problem 3^o. On the front of the spherical detonation we have $\lambda = a_1^2/c^2 =$ constant. The point $z_1 = a_1^2/c^2$, $V = 0$ corresponds to the external side of the jump. The value of all the characteristics on the internal side of the jump will be determined when the follow-

ing are assigned: the density ρ_1 and pressure p_1 in the quiescent mixture, the heat of reaction $q_1 - q_2$ that satisfies the inequality (3.21), and the velocity of propagation of the shock wave c. From the conditions at the jump two solutions corresponding to the points M and N (see Fig. 3) are obtained.

Continuation of motion from the point M is accomplished by the integral curve MA (see Fig. 8). The point A corresponds to the boundary of the expanding nucleus in which the gas - the product of the chemical reaction accompanying the detonation - is at rest.

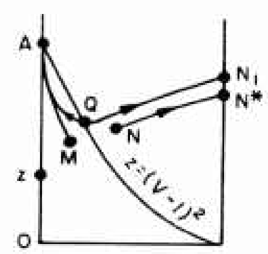

Fig. 8

Continuation of motion from the point N is accomplished by the integral curve NM^*. It is obvious that it is impossible to continue this motion to the center of symmetry. Motions of such character may be excited by means of the additional spherical piston which we considered in Problem 2°.

The velocity of propagation of the detonation at a given value $q_1 - q_2$ is a minimum when the points M and N converge into a single point Q located on the parabola $z = (V-1)^2$. In this critical case

the motion can be continued either to the right at the point N_1^* or to the left to the point A. In the first case, pressures greater than at the jump are obtained in the internal region and on the piston; in the second case, beyond the jump a rarefaction wave arises in which the pressures diminish from the point Q up to the point A.

Various authors paid much attention to the problem of determining the detonation velocity.[6] We can point out that all hydrodynamic conditions are satisfied if a jump takes place at the point M and the motion continues along the integral curve MA.

Experimental data show that there is a regime of detonation corresponding to a minimum velocity satisfying the point Q (see Fig. 8), with subsequent change of characteristics along the integral curve QA. For substantiation of this fact, Zeldovich drew upon some reasonings connected with the chemical mechanism of the reaction in the shock-wave front.

It is obvious that at any moment each motion of the type under consideration is mechanically similar to any state, and, in particular, to the state for the moment of time, as close as desired to the initial moment of detonation at the center of symmetry. This circumstance can serve as a reason for the choice of velocity of detonation from the data on the method of excitation in the center of symmetry. If the detonation is initiated in the center by external pressure,

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[6] For example, see Zeldovich [4] and Grib [5].

then a point of type N must correspond to the jump. On the other hand, the demand that the velocity of the products of reaction at the center of symmetry should equal zero leads to solutions in which a point of type M corresponds to a jump. Simultaneous fulfilling of these two conditions can be satisfied only for points of type Q and an integral curve of the type QA. These considerations may be considered as a certain basis for the fact that in practice a regime of jumps corresponding to points of the type Q must be obtained.

Received by the editor
28 IV 1945

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